A STUDY OF MIDDLE SCHOOL STUDENTS' MATHEMATICAL PROFICIENCY IN THE MATHEMATICS CLASSROOM

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Abstract

This study investigated middle school students' mathematical proficiency in the mathematics classroom featured in conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition among six hundred Grade (8) students in Yangon Region. A descriptive research design was adopted and two types of instruments: a mathematical proficiency test reflected in former four strands and a productive disposition questionnaire towards mathematics were employed. The internal consistency reliability coefficient for the test was (.714) and that for questionnaire was (.704). The collected data were analyzed using descriptive statistics of frequency, percentage, mean, and standard deviation and inferential statistics of Pearson product-moment correlation. Research findings revealed that most of the students had moderate level of mathematical proficiency. Moreover, the results showed that students' procedural fluency was the highest and strategic competence was the lowest among the former four strands. There was a significant positive relationship among five strands of mathematical proficiency. Grounded on this baseline study, it was thus, suggested that first, a nationwide survey on mathematical proficiency at all grades should be conducted and then, intervention program should be mapped out to cater students' current level.

Keywords: Mathematical Proficiency, Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning, Productive Disposition

Introduction

In this age, routine is different. The world is gradually filled with more technological advancements at an alarming rate. However, there is no exaggeration that all those expansions concern somewhat with mathematics. Thus, the idea that mathematics every person needs is to be able to execute solely basic computations has been old-fashioned. Rather, today society members require the ability to have greater understanding of mathematical ideas, use mathematical reasoning and logic, and solve many problems to adapt to those changes. In the same vein, they all need to have increased mathematical proficiency so that they will meet current and future demands of society.

Ally (2011) pointed out the importance of background mathematical knowledge for all students. According to him, if one has flaws in his or her mathematical background knowledge or lacks a solid grasp of facts, procedure, definitions, and concepts of mathematics, he or she will significantly be handicapped in mathematics. Along with this, every student's idea on any domain of mathematics is shaped by his or her experiences in touch with the subject at all levels passed. National Council of Teachers of Mathematics (NCTM, 2000) supported this view that students' understanding of mathematics, their ability to use it to solve problems, and their confidence in, and disposition toward mathematics are all shaped by the teaching they had encountered in school.

Sadly, current teaching situations in Myanmar traditionally focus on rote learning (CESR, 2013). It aims to have students reproducing content, no matter if they make sense or not. Put the

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other way, it does not consider students' actual understanding and own thinking, instead, forgoes on replicating mathematical ideas. When the individuals memorize something that they do not fully understand, they cannot construct any link between the new information and the stored one (Ausubel, 1968). That is why students cannot connect different bits of mathematical ideas in their brain and preserve them last longer.

Likewise, they cannot utilize them in solving many problems. Also, they cannot determine whether the computational procedure is appropriate or not. As a result, their motivation towards mathematics leads to decline at a certain level and so, they can get more difficulties in learning mathematics. In other words, their level of mathematical proficiency may be lower as much as possible. To cherry-pick this situation, there is an urgent need to determine students' level of mathematical proficiency.

Moreover, if the diagnostic result shows a poor outcome, something better can still be done. There were many instances confirmed that such assessment had positive great impact on students' progress in mathematics education. As an example, the result from National Assessment of Educational Progress (NAEP) in 1999 underlined that only 21% of Grade (4) students, 24% of Grade (8) students, and 16% of Grade (12) students are nationally proficient in mathematics. Teachers, principals, parents, policy makers, and researchers all had used this result in developing ways to improve mathematics education in the U.S. As a result, all these graders scored higher than in recent previous assessment. For example, in 2009, 26% of twelve-graders performed at or above the proficient level. That is why there is a need to diagnose students' mathematical proficiency.

Basically, it is undeniable that today society requirement, current mathematics teaching orientations, and the impact of NAEP assessment on American mathematics education were the primary derives to undertake this study.

Purposes of the Study

The present study generally tends to investigate middle school students' mathematical proficiency in the mathematics classroom. Specifically, it aims to

- investigate students' mathematical proficiency in mathematics,
- compare former four strands of mathematical proficiency among students,
- find out the relationship among five strands of students' mathematical proficiency, and
- make suggestions for promoting five strands of mathematical proficiency of the students.

Research Questions

This study sought to address the following questions.

Q_1: To what extent do the students possess mathematical proficiency in mathematics?

Q_2: Which strands are the highest and the lowest among former four strands of mathematical proficiency?

Q_3: Is there any significant relationship among the strands of mathematical proficiency of the students?

Scope of the Study

Even though this research reached its goals, there were some unavoidable limitations. First, because of the time limit, this study was conducted on a small size of middle (Grade 8) students came from eight selected basic education high schools only in Yangon Region within the academic year (2018-2019). Second, this study is concerned only with mathematical proficiency reflected in five strands (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) due to the 2001 consensus report by National Research Council (NRC). Finally, the content area to be studied is limited to number domain. There are many reasons of being this domain targeted. First of all, Kilpatrick et al. (2001) suggests that number sense is the foundation of all later number work. Moreover, number is a basis to describe and understand the world. In addition, every mathematics curriculum during all school years is not outside the number domain. Furthermore, this domain supports other branches of mathematics like algebra, geometry, probability and statistics and vice versa. For example, a better understanding of number basis would enable students to handle algebraic operations and manipulation stronger (Watson, 1990).

Theoretical Framework

Kilpatrick and his colleagues coined the term mathematical proficiency with five strands as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (Kilpatrick et al., 2001) and the following concepts hereby provide the bases for this study:

Conceptual understanding is defined as an integrated and functional grasp of mathematical ideas like concepts, operations and procedures (Kilpatrick et al., 2001). Such understanding allows students to build new knowledge through making connections with the previously learned knowledge. It promotes retention and fosters the development of fluency. Therefore, students with conceptual understanding truly know more than isolated facts and methods. They understand why a mathematical idea is important and the kinds of contexts in which it is useful. Also, they have the ability to represent different mathematical situations and to connect these representations.

Procedural fluency means ‘skills in carrying out procedures flexibly, accurately, efficiently, and appropriately’ (Kilpatrick et al., 2001). So, students displaying procedural fluency know procedures, and when and how to use them appropriately, and can apply them flexibly, accurately and efficiently. It is worthy for teachers to note that students’ learning and practicing procedures should be based on understanding in that those who learn procedures without understanding can typically do no more than apply the learned procedures, whereas ones who learn with understanding can modify or adapt procedures to make them easier to use.

Strategic competence refers to ‘the ability to formulate mathematical problems, represent them, and solve them’ (Kilpatrick, et al., 2001). By the same token, this strand is generally concerned with a person’s ability to formulate a problem mathematically, and then use his or her previous knowledge to solve it. Having strategic competence enables a person to make out which strategies may be useful and appropriate in solving the problem. Hence, a student with strong strategic competence is able to have several approaches to the solution of a problem and then, choose flexibly among them through reasoning and reflecting on his or her experiences. Rather, students who do not possess adequate strategic competence will approach a mathematical problem through a trial and error strategy frequently.

Adaptive reasoning stands for the capacity to think logically, reflect, explain and justify one's answer (Kilpatrick et al., 2001). It is the glue that holds everything together, the lodestar that guides learning' in that adaptive reasoning allows for concepts and procedures to connect together in sensible ways, suggests possibilities for problem-solving, and allows for disagreements to be settled in reasoned ways. More specifically, it includes not only formal proofs and deductive reasoning, but also informal explanations or justification about mathematical ideas, intuition and inductive reasoning based on patterns, analogy and metaphor. Therefore, students using adaptive reasoning can think logically about the relationships among concepts and situations, consider appropriate alternatives, reason correctly and justify the conclusions.

Productive disposition is the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics (Kilpatrick et al., 2001). Rather than seeing mathematics as a set of arbitrary rules that one must memorize, students with productive disposition view mathematics as a system of connected conceptions that can be understood with perseverance and diligent effort. Students with strong productive disposition are confident in their knowledge and ability, they see that mathematics is reasonable and understandable, and believe that appropriate effort and experiences makes them achieved mathematics. Then, they believe that mathematics is for everyone and reject the mathematics mystery. Moreover, Siegfried (2012) used eight constructs: affect; beliefs; goals; identity; mathematical integrity; motivation; risk taking; self-efficacy appropriately in attempts to define the term 'productive disposition'.

In addition, Kilpatrick and his colleagues (2001) also noted that these five strands are interwoven and interdependent in the development of mathematical proficiency. Besides, these five strands provide a framework for discussing the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency which enables students to cope with the mathematical challenges of daily life and also enables them to continue their study of mathematics in high school and beyond.

Research Method

Research Design

The research design used in this study was a descriptive design under quantitative approach.

Instruments

To address the above research questions, two instruments were used. They include a mathematical proficiency test composed of former four strands and a questionnaire about productive disposition towards mathematics. The test questions were adopted from standardized question banks (TIMSS, NAEP) with a little modification to align with Myanmar mathematics curriculum. Items in the questionnaire were developed on the basis of the definition of productive disposition by Siegfried (2012).

Afterwards, to attain the reliable data, expert validation was conducted through the careful assessment of six experts who are very special in mathematics education and teaching from Department of Methodology in both Yangon and Sagaing Universities of Education. Thereafter, making necessary changes will be carried out under the consultation of the supervisor.

Pilot Testing

A pilot test was administered on November 25, 2018 to (40) Grade Eight students in No. (7), Basic Education High School in Alone. To measure the reliability of the instrument, the Cronbach's Alpha was calculated. This process gave rise to the internal consistency reliability coefficient (.714) for mathematical proficiency test and (.704) for questionnaire.

Population and Sample

There were (600) Grade Eight students coming from Yangon Region involved as participants in this study (see Table 1). Moreover, the equal-sized (non-proportional) random sampling technique was utilized.

Table 1 Population and Sample Size

No.	Township	School	No. of Participant	
			Population	Participant
1	North Dagon	BEHS 3	492	75
2		BEHS 5	254	75
3	Dagon	BEHS 1	696	75
4		BEHS 2	237	75
5	Thanlyin	BEHS 1	452	75
6		BEHS 4	186	75
7	Mingaladon	BEHS 3	440	75
8		BEHS 2	260	75
	Total			600

Note: BEHS = Basic Education High School

Data Collection

The modified instrument was distributed to all participants of the eight sample schools with the help of the headmaster/headmistress of those schools in December, 2018. Then, all data will be collected, and entered into the computer data file.

Data Analysis

Once the data were collected, both mathematical proficiency test and questionnaire were coded using the Statistical Package for the Social Science (SPSS). Then, the data were analyzed by using descriptive statistics. In order to measure the level of students' mathematical proficiency, mean, standard deviation, frequency and percentage were used. Furthermore, Pearson product-moment correlation was employed to determine the relationship among the strands of mathematical proficiency.

Research Findings

Findings about Mathematical Proficiency among Students

A total score was computed from the five strands of mathematical proficiency. Then, it was found that (a) the scores were ranged from (67) to (153), and (b) the sample mean for eight selected schools was (111.66) with its standard deviation (15.01). With respect to those results, the level of mathematical proficiency was sorted into three categories: poor (scores below 96.65), moderate (between 96.65 and 126.67 both inclusive), and high (scores above 126.67).

Afterwards, as can be seen in Table (2), 15.17% of the students (N = 91) got in touch with poor level, 69.83% of the students (N = 419) with moderate and 15% of the students (N = 90) with high levels respectively.

Table 2 Students' Level of Mathematical Proficiency

Mathematical Proficiency	**No. of Student**	**Level**		
		Poor (%)	**Moderate (%)**	**High (%)**
	600	15.17	69.83	15

Findings about Mathematical Proficiency in Former Four Strands

A total score for the first four dimensions was calculated separately from the group of items under each strand whereas the full score were fixed as (13) points for every aspect. Every single group includes (7) items respectively with (5) multiple choices, one short-response and one long-response.

Then, it got a message that (a) on conceptual understanding, the score ranged from (0) to (13), and the sample mean was (6.13) with standard deviation (2.54), (b) on procedural fluency, the score ranged from (0) to (13), the sample mean was (6.34) and the standard deviation was (3.40), (c) for strategic competence, the score ranged from (0) to (12), the sample mean was (4.41) and the standard deviation was (1.81), and (d) for adaptive reasoning, the score ranged from (0) to (13), the sample mean was (5.52) with standard deviation (2.76).

Manipulating these results yields students' mathematical proficiency for each strand to three categories: poor, moderate, and high. Students with scores above the (+1) standard deviation from the sample mean came up with high level and those with scores below the (-1) standard deviation from the sample mean were at the poor level. Then, the students with the scores between (+1) and (-1) standard deviation from the sample mean were fallen in the category with moderate level. Table (3) pinpoints students' mathematical proficiency in five strands.

Table 3 Students' Level of Mathematical Proficiency in Former Four Strands

Level	**Mathematical Proficiency in Former Four Strands (%)**			
	Conceptual Understanding	**Procedural Fluency**	**Strategic Competence**	**Adaptive Reasoning**
Low	16	16	16.33	14.67
Moderate	72.67	70.17	75.67	71.83
High	11.33	13.83	8	13.5

Along with these findings, it was significantly to be found that only 17.3% of the students (N=104) got the mean value (0.17) in item one among five multiple choices and 82% of the students with score under mean value i.e. zero chose the option B.

To measure the extent of productive disposition, five-point Likert scale items were used. The total score (percentage) was performed from the eight constructs together with (3) items each which all comprises productive disposition towards mathematics. Then, it was noticed that 66.40% of the students (N=398) had positive productive disposition, 19% of the students (N=114) had negative productive disposition and the rest 14.60% of the students (N=88) had neither (see Table 4).

Table 4 Students' Degree of Mathematical Proficiency in Productive Disposition

Productive Disposition	No. of Student	Level of Agreement (%)		
		Positive	Neutral	Negative
	600	66.4	19	14.6

Findings about Comparison for Former Four Strands of Mathematical Proficiency

With respect to mean scores in former four strands: conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning, it was noted that the highest mean value was (6.42) equals to that of procedural fluency subscale and the lowest was (4.42) refers to that of strategic competence subscale, too. Table (5) shows the comparison of mean scores with their respective standard deviation.

Table 5 Comparison of Mean Scores for the Former Four Strands

Strand of Mathematical Proficiency	No. of Student	Mean	Standard Deviation
Conceptual Understanding (CU)	600	6.13	2.54
Procedural Fluency (PF)	600	**6.34**	3.40
Strategic Competence (SC)	600	**4.41**	1.81
Adaptive Reasoning (AR)	600	5.52	2.76

Findings about Relationship among the Five Strands of Mathematical Proficiency

To determine the interrelationship among five strands of mathematical proficiency, Pearson product-moment correlation was used. According to Gay and Airasian (2003), the correlation coefficient less than plus or minus (.35) was interpreted as low or no relation, between plus or minus (.35) and (.65) as moderate relation and higher than plus or minus (.65) as high relation. Then, as can be seen in Table (6), there was a significant positively moderate correlation among the five strands of mathematical proficiency.

Table 6 Correlation among Five Strands of Mathematical Proficiency

Correlation					
	CU	**PF**	**SC**	**AR**	**PD**
CU	1	.584**	.552**	.537**	.375**
PF		1	.561**	.561**	.418**
SC			1	.512**	.424**
AR				1	.374**
PD					1
Correlation is significant at the 0.01 level (2-tailed)					

Note: CU = Conceptual Understanding, PF = Procedural Fluency, SC = Strategic Competence, AR = Adaptive Reasoning, PD = Productive Disposition

Discussion, Suggestions and Conclusion

Discussion

This section attempts to discuss serious findings about the three research questions framed for this study.

Discussion for Research Question One

The results relating to the research question one: To what extent do the students possess mathematical proficiency in mathematics? exhibited in Table (2) indicates that most of the students (69.83%) from the selected schools had moderate level of mathematical proficiency with 15.17% poor level of mathematical proficiency and only just few students (15%) showed high level of mathematical proficiency.

Moreover, it is apparent that this finding is a bit different with the result from the Awofala's research conducted in Nigeria, 2017. Evidence from his research revealed that most of senior secondary school students from the elitist schools had high levels of mathematical proficiency (Awofala, 2017). Personally, this disparity may be due to one possibility. According to the results in Table (3), it was found that most of students possess moderate level of ability in all four strands with mostly strong productive disposition. That is why most of the students had moderate level of mathematical proficiency in that all those strands had a great impact on their mathematical proficiency. In other words, all those strands represent the comprehensive term 'mathematical proficiency'.

Like an evolution process, those results in all former strands may be due to the effect of teaching-learning process adopted in the classroom. Traditionally, mathematics education in Myanmar put more emphasis on rote learning (CESR, 2013) which less emphasizes on providing students opportunities for learning though as NRC (2001) pointed, teaching-learning should be enactment, on the mutual and interdependent interaction of three elements: mathematics content, teacher, and student.

Significantly, in item one which assesses procedural fluency about order of operation only 17.3% of the students got the correct answer. This means that most of the students cannot add and multiply numbers in a right procedure. Moreover, 82% of the students chose Option B as the correct answer in this item. This exhibits that most of the students had misconception in the process of four basic operations. This additional result is similar with the findings of Moodley (2008) that misconception influenced the achievement of the students on procedural fluency. This may be due to the fact that teaching procedural fluency does not ground on sound conceptual understanding.

Discussion for Research Question Two

The results relating to the research question two: What are the highest and the lowest among former four strands of mathematical proficiency? exhibited in Table (5) indicates that the mean score of procedural fluency was the highest but on the other hand, that of strategic competence was the lowest. It means that most of the students in this study outperformed in procedural fluency rather than other strands. In other view, facility in computation at a higher degree, did not lead the students to develop other strands completely i.e. skill in procedural fluency is not to be counted into the development of other strands.

This result is also similar with the finding of Wu (2008) in China that Chinese students' procedural fluency was at a higher level compared to other strands: conceptual understanding and word problem. From personal point of view, this may be associated with traditional teaching method of mathematics held in the classroom. Samuelsson (2010) showed that there were no significant differences between traditional and problem solving teaching methods when assessing procedural fluency but students' progress in conceptual understanding, strategic competence, and adaptive reasoning was significantly better when teachers taught with a problem-based curriculum. In other words, it indirectly highlights that current traditional mathematics teaching methods weightage on practicing students' procedural fluency.

Discussion for Research Question Three

The results in Table (6) which attempts to answer the research question three: Is there any significant relationship among the strands of mathematical proficiency of the students? indicate that there was a significant moderate correlation at $p < 0.1$ among five strands of mathematical proficiency. Since a positive linear correlation was found, it can be concluded that (a) the strands are significantly correlated with each other and (b) when one is high, the others will be high and while one is low, there will be the others low.

This finding is supported by the literature explained by National Research Council (NRC, 2001). As a student gains conceptual understanding, computational procedures are remembered better and use more flexibly to solve new problems. In turn, as a procedure becomes more automatic, he is enabled to think about other aspects of a problem and to tackle new kinds of problems, which leads to new understanding. Solving challenging problems develops new understanding and fluency. Moreover, adaptive reasoning is the glue that holds all strands to be a network. It states that this strand is interrelated with other strands and vice versa. Students without proper developing the four strands prescribed will not engage in mathematics tasks in long-lasting and in turn, students without proper productive disposition will do so.

Suggestions

The mathematics to which students are exposed from preschool to Grade Eight has many aspects. They have already learned many things about number for at least eight years at school. But, according to the findings of this study, most of the sample students had moderate level of mathematical proficiency and even in procedural fluency, misconception influenced the students' achievement. This underlines that such proficiency requires to be developed in right manner. Actually, the ways in which mathematical proficiency is developed may be a pedagogical challenge for most of mathematics teachers. So, with the aim of developing mathematical proficiency at least with misconceptions, the following points are suggested in accordance with the related literature.

1. Teaching and learning should be the product of trilateral interaction among three elements: teacher, content and students rather than relying on teacher.
2. A mathematics curriculum should be coherent, focused on many important mathematics ideas from different areas, and well-articulated across the grades.
3. The classroom practices should give students the opportunities to develop mathematical proficiency in five strands.

4. The opportunities to develop conceptual understanding should place more emphasis on connection of many mathematical ideas through reasoning and justifying.
5. The opportunities to develop procedural fluency should stem on methodical, well-timed practices using different mathematical operations but rooted in conceptual understanding.
6. The opportunities to develop strategic competence should rely on frequent exposure to many mathematical problems that reflect real-world situations and focus on choosing the appropriate problem-solving strategy to the mathematical situation.
7. The opportunities to develop adaptive reasoning should emphasize on encouraging to actively engage in justification.
8. The opportunities to develop productive disposition towards mathematics should make a focal point on demonstrating sensitivity towards learner's previous difficulties, encouraging persistence, and accepting mistakes as part of learning.

Conclusion

According to National Mathematics Advisory Panel (2008), a strong foundation in high school mathematics through Algebra II is strongly correlated with access to college, graduation from college, and earning in the top quartile of income from employment. It highlights the importance of mathematical proficiency or experiences students took with them. Therefore, this study was conducted for the purpose of studying middle school students' mathematical proficiency in the mathematics classroom. The descriptive survey method was utilized. To gather the necessary data for this study, two measuring tools: mathematical proficiency test reflected in four parts and productive disposition questionnaire were used whereas test items came from the NAEP and the TIMSS study. Six-hundred middle students in Yangon Region during the school year 2018-19 were involved as respondents.

The analysis was structured in accordance with the five strands of mathematical proficiency. To determine the level of students' mathematical proficiency, the data from the test items were analyzed by using a Statistical Package for Social Science (SPSS) and the questionnaire was analyzed thematically. Then, the findings of this study are summed up in line with three research questions as follows.

1. Most of the students from the selected schools had moderate level of mathematical proficiency but with misconception in procedural fluency.
2. Most of the students from the sample schools outperformed in procedural fluency than other strands.
3. There was a significant positive moderate correlation among five strands of mathematical proficiency.

Actually, what kind of instruction given by the teacher in the classroom also affects the development of students' mathematical proficiency. Moreover, the instruction is in context (cited in Adding It Up, 2001). This means that mathematical proficiency cannot be achieved through isolated efforts. All interested stakeholders have to work together to improve mathematics at school. Furthermore, based on the limitations, findings and suggestions sections, some of the following recommendations for further studies can be underscored as follows.

1. Conducting this study only at middle school level, there should be further studies at other levels in that it takes time to develop mathematical proficiency.

2. Restricting this inquiry only in Yangon Region, further studies should be carried out in other regions for replication.
3. Being the study area in number domain, there should be investigated in other domains because mathematics curriculum has not been confined to this area only.
4. Framing this research only in survey within a short duration, further studies should be in qualitative such as opportunities to develop students' mathematical proficiency.
5. With the aim to improve students' proficiency in mathematics further studies should be concerned with instructional practices that promote mathematical proficiency.
6. In order to overcome the teachers on the danger of misconception, additional studies should explore the factors that influence students' mathematical proficiency for all grades.

As a significant factor, it can be expected that this study can help the teachers and many curriculum developers to take the results: students' current mathematical proficiency from this study as a beginning point in their teaching or reforming curriculum.

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